Energy minimization for self-organized structure formation and actuation

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(Received 14 November 2006; accepted 18 January 2007; published online 22 February 2007)

An approach for creating complex structures with embedded actuation in planar manufacturing steps is presented. Self-organization and energy minimization are central to this approach, illustrated with a model based on minimization of the hyperelastic free energy strain function of a stretched elastomer and the bending elastic energy of a plastic frame. A tulip-shaped gripper structure illustrates the technological potential of the approach. Advantages are simplicity of manufacture, complexity of final structures, and the ease with which any electroactive material can be exploited as means of actuation. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2695785]

Electromechanically active polymers.[1–5] are lightweight materials that convert electrical into mechanical energy. They are ideally suited for artificial muscles in biomimetic robots.[6] The design of active structures (robots) based on actuators follows an approach in which a passive skeletal structure composed of rods and joints is augmented by actuators, which apply force to the skeleton, such that it changes its shape. We present an approach to robot design that takes advantage of a principle of structure formation based on the minimization of free energy. It relies on planar manufacturing steps only: a prestretched elastomer sheet is combined with an elastic frame, resulting in complex out-of-plane structures.

An elastomer consists of a highly entangled and cross-linked network of polymer chains [Fig. 1(a)] with large configurational entropy, and thus low free energy. Upon stretching of the elastomer, the chains are elongated, thus lowering the configurational entropy and raising the free energy [Fig. 1(b)]. The stretched elastomer combined with an elastic frame gives rise to complex structures [Fig. 1(c)], because the energy released from the contracting elastomer is partially stored as bending energy in the frame. The simultaneous minimization of these two energy contributions provides the driving force towards the establishment of a unique, complex structure. The addition of an external stimulus allows the self-organized system to change its configuration, and thus to actuate [Fig. 1(d)]. This can be achieved with a third energy contribution, which couples the elastic energy of the frame with the free energy of the elastomer, such that a new energy minimum with a different structural configuration is established.

The proposed technology platform rests on the principle of minimal surfaces[7] found everywhere in nature, for instance, in the highly complex shape of soap films trapped within fixed boundaries. The beauty of such structures is evident, serving as toys for children, as well as inspiration for artists and architects.[8]

The principle of complex structure formation and actuation due to minimization of free energy is substantiated with a model actuator, shown in Fig. 2. It consists of a sheet of stiff plastic bent into a shell, to the ends of which a sheet of elastomer is attached. Compliant electrodes are applied to both sides of the elastomer sheet, forming an elastomeric capacitor (dielectric elastomer actuator) driven by Maxwell stress.[9] Actuators based on this principle are known to undergo large actuation strains.[1]

The analytical model of this structure is based on the energy stored in the bending beam, U_{beam}, and the free energy of the elastomer, U_{elast}. The energy in the freely jointed chain model in its Gaussian limit[10] is

$$U_{\text{elast}}(\lambda_1) = x_1' x_2' x_3' \frac{1}{2} G(\lambda_1^2 + \lambda_1^{-2} - 2), \tag{1}$$

where x_1', x_2', and x_3' are the initial length, width, and thickness of the rubber band, respectively, G is the shear modulus, and λ_1 is the deformation ratio in the length direction. It is assumed that there is no deformation in width ($\lambda_2 = 1$, pure shear) and that the volume is constant ($\lambda_1 \lambda_3 = 1$, corresponding to a Poisson ratio of $\frac{1}{2}$). As discussed in Ref. 10, the Gaussian model provides reasonable agreement with most

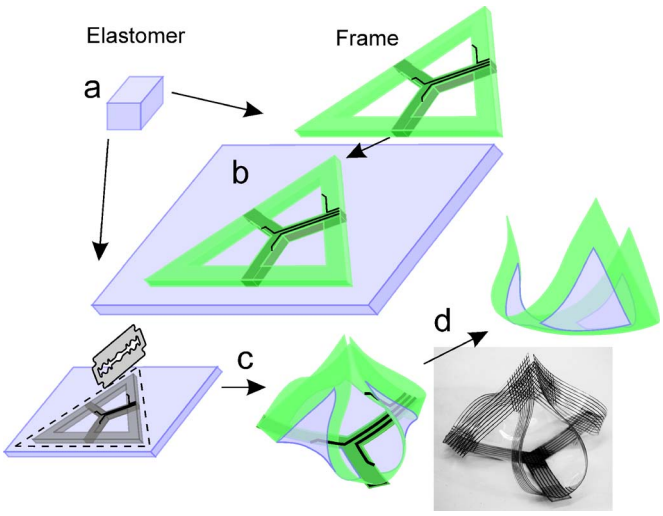

FIG. 1. (Color online) (a) Prestretched elastomer is combined with a frame (b), in a planar process (c). Release results in large deformations through the simultaneous release of entropic energy and storage of bending energy in the frame. (d) Additional energy contribution results in actuation.

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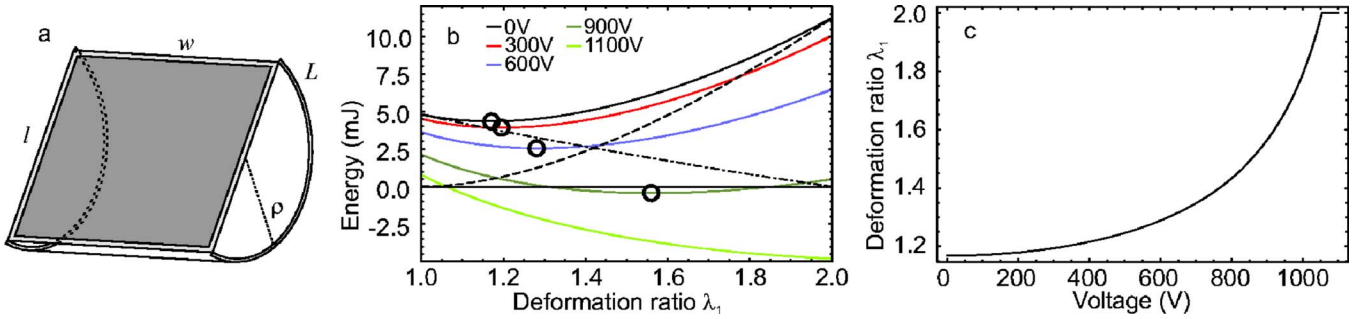

FIG. 2. (Color online) (a) Plastic frame shell dielectric elastomer actuator deforms by opening when a voltage is applied. (b) Energy contributions (dash: elastomer, dash dot: frame) are plotted as a function of the deformation ratio of the elastomer sheet, the full lines corresponding to the total energy at different applied voltages. The minimum in the energy (circles) shifts toward higher deformation ratios when the voltage is increased. (c) Deformation ratio is plotted as a function of voltage, showing that the actuator becomes flat at 105 V/μm.

rubber experiments up to strain levels of 70%–100%.

The separation of the tips of the bending beam l equals the current length of the rubber film, $l=x_1'\lambda_1$; therefore λ_1 will be employed as the independent variable. Further, the energy of the bending beam is

$$U_{\text{beam}}(\rho)=\frac{Ebh^3}{24}\frac{L}{\rho^2}, \tag{2}$$

where b is the width, h the thickness, L the length of the beam, E Young's modulus of the plastic, and ρ the radius of curvature, where $l=2\rho\sin(L/2\rho)$, provided $l>0$.

Actuation is achieved by providing an external energy stimulus that couples the energy of the bending beam and the elastomer. Here, the free energy of the elastomeric capacitor in the voltage-controlled case depends on the applied voltage and on the current state of deformation of the elastomer,

$$U_{\text{cap}}(\lambda_1,V)=-\frac{x_1'x_2'}{x_3'}\frac{\epsilon_r\epsilon_0}{2}V^2\lambda_1^2, \tag{3}$$

where $\epsilon_r\epsilon_0$ represents the absolute permittivity of the elastomer and V the applied voltage.[11]

In Fig. 2(b), the total energy of such an actuator is evaluated.[12] The length of the bending beam was chosen to be twice that of the initial length of the rubber film, $L=2x_1'$. The energy of the bending beam (dash-dot curve) drops when the deformation ratio increases and vanishes when the beam is flat at $\lambda_1=2$. The free energy of the rubber (dashed curve) increases with deformation and vanishes in the undeformed state, $\lambda_1=1$. The equilibrium length of the rubber film is found where their sum total (heavy unbroken curve) has a local minimum. Thus, the local energy minimum corresponds to a unique shape of the structure, and we therefore term this type of structure a minimum energy structure.

The application of a voltage [see Eq. (3)] shifts the energy minimum towards a higher deformation ratio. In Fig. 2(b) the total energy is plotted for different voltages. At 1100 V no minimum is found, hence the bending beam is flat. The energy minimum is plotted versus applied voltage in Fig. 2(c), showing nonlinear increasing deformation, typical for elastomer actuators. At 1054 V, corresponding to an electric field of 105 V/μm, the structure becomes flat, and no further actuation is possible. The presence of a voltage at which the actuator is fully actuated is significant as it allows definition of a structural maximum voltage below breakdown. Thus, minimum energy actuators can be designed to operate without danger of dielectric breakdown.

The suggested minimum energy structures constitute a general approach to the understanding of complex structure formation and actuation phenomena. The actuation principle chosen here does not present the only approach to large strain actuation of polymeric materials. Electrically stimulated large strain actuation has been demonstrated in conducting polymers,[2] electrolytic actuators based on carbon nanotubes,[3] ferroelectric terpolymers,[4] and elastomer composites[5] among many others. Such materials could replace either the elastomer or the frame to provide for a coupling between the structure and energy. Thermally stimulated liquid crystalline elastomers[13] and magnetically actuated gel composites[14] could also be incorporated.

A separate "leaf" was cut from the structure of Fig. 1 and compliant electrodes were applied (see Ref. 15 for materials). The leaf was mounted on an elevation stage, such that the tip was free to move in the vertical direction. The tip was made to push downwards on a balance (Chyo JL-200, 0.1 mg resolution), such that a force-voltage curve (Fig. 3) and a force-displacement curve could be measured (inset of Fig. 3). The maximum force corresponds to 700 mg of weight, with a force constant of 1.0 N/m found from the slope. At 2.0 kV, the leaf lifted itself from the measuring tip, corresponding to an actuation movement higher than 11 mm.

An application of the minimum energy structure of Fig. 1 as a gripper is illustrated in Fig. 4. In the actuated state (here 3.0 kV), the structure opens to envelop large objects. When the voltage is removed, the structure contracts and the

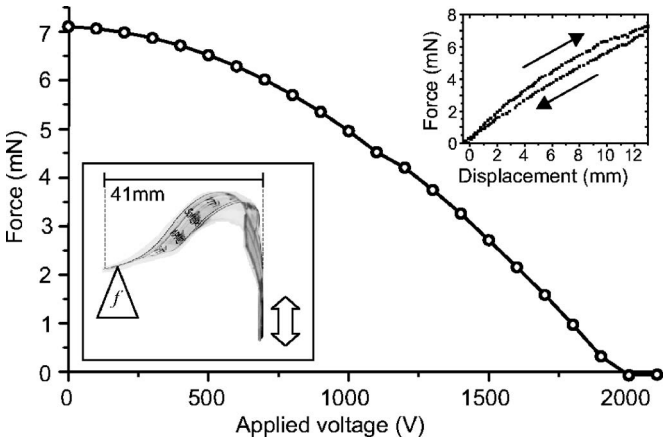

FIG. 3. Force measurement on one of the "leaves" (left inset). Force-voltage curve, obtained at a bias force pushing the tip back 11 mm, and force-displacement curve (right inset).

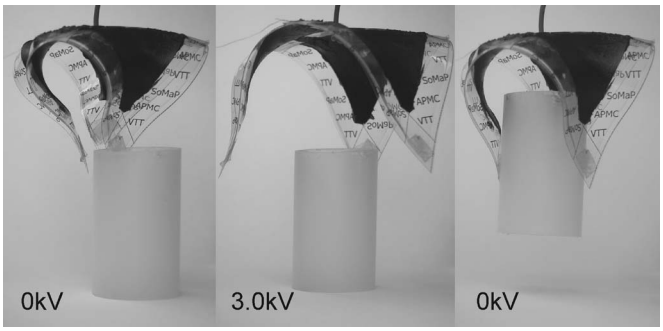

FIG. 4. Minimum energy structure of Fig. 1 provides gripper action. It opens when a voltage of 3.0 kV is applied, enough to grasp objects when the voltage is removed.

three claws deliver a balanced holding force to the object in its center, enough to carry the object.

The concept of self-organized minimum energy structures presented above is constantly used in nature, as exemplified by nastic structures, where changes in the osmotic pressure (free energy) results in large motions.[16] Another example is the structure of microscopic biological organisms (viruses and cells) which could be explained either with a stress[17] or an energy approach.[18] The presented macroscopic objects are analogous to such microscopic objects, as all attain their shapes through minimization of structural energy.

In summary, complex minimum energy structures are formed by combining energy and entropy elasticity in bending frames and stretched elastomers. Such minimum energy structures are capable of actuation, with potential as self-organized robots through extremely simple manufacturing steps.

This work was supported by the German BMBF, the Finnish Tekes, and the Austrian Science Funds within their respective national nanoinitiative programs. One of the authors (G.K.) acknowledges financial support from the European Commission through a Marie Curie Intra-European Fellowship. Two of the authors (G.K. and W.W.) thank Reimund Gerhard(-Multhaupt) for his support.

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[11] In the charge-controlled actuator the total free energy is $U(Q)=U_{\text{beam}}+U_{\text{elast}}+Q^2/2C$, where C is the capacitance of the element. The voltage-controlled case is obtained via a Legendre variable transformation, $U(V,\cdots)=U(Q,\cdots)-QV=U_{\text{beam}}+U_{\text{elast}}-CV^2/2$.
[12] The following parameters were chosen: $E=1$ GPa, $G=100$ kPa, $L=b=2x_1'=x_2'=100$ mm, $h=200$ μm, $x_3'=20$ μm, and $\epsilon_r=3$.
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